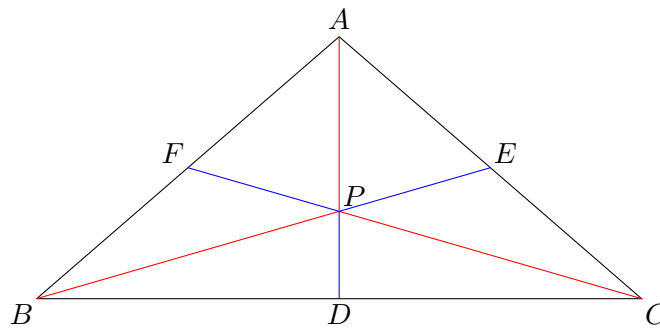


LUCA puzzles 2024: Puzzle 60

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Let $\triangle ABC$ be an equilateral triangle of size a . Let P be any point inside the triangle.



Let $PD = x$, $PE = y$, $PF = z$ be the perpendiculars to the sides \overline{BC} , \overline{AC} and \overline{AB} .

Area of $\triangle ABC$

$$\begin{aligned} A_{\triangle ABC} &= \frac{ax}{2} + \frac{ay}{2} + \frac{az}{2} \\ &= \frac{a(x + y + z)}{2} \end{aligned} \tag{1}$$

At the same time, area of an equilateral triangle

$$A_{\triangle ABC} = \frac{ah}{2} \tag{2}$$

Equating (1) and (2), we get

$$h = x + y + z$$

That means, if we take *any* interior point P inside the equilateral triangle $\triangle ABC$, the sum of the three perpendiculars to the sides will be equal to the altitude.